

# A possible anthropic solution to the Strong CP problem

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## Abstract

We point out that the long-standing strong CP problem may be resolved by an anthropic argument. The key ideas are: (i) to allow explicit breaking(s) of the Peccei-Quinn symmetry which reduces the strong CP problem to the cosmological constant problem, and (ii) to conjecture that the probability distribution of the vacuum energy has a mild pressure towards higher values. The cosmological problems of the (s)axion with a large Peccei-Quinn scale are absent in our mechanism, since the axion acquires a large mass from the explicit breaking.

## I. INTRODUCTION

One of the profound problems of the standard model (SM) is the strong CP problem. In the quantum chromodynamics (QCD), there is no a priori reason to forbid the following CP-violating operator,

$$\mathcal{L} = \frac{g_s^2 \theta}{64\pi^2} \epsilon_{\mu\nu\rho\sigma} G^{(a)\mu\nu} G^{(a)\rho\sigma}, \quad (1)$$

where  $G_{\mu\nu}^{(a)}$  is the field strength of the  $SU(3)_c$  gauge fields, and  $g_s$  is the  $SU(3)_c$  gauge coupling. This operator contributes to the electric dipole moment of the neutron, and the experimental measurements have severely limited the parameter  $\theta$  as  $|\theta| < 10^{-(9-10)} \equiv \theta^{(\text{exp})}$  [1]. Such a tight constraint on  $\theta$  is regarded as a fine-tuning; this is the strong CP problem.

The Peccei-Quinn (PQ) mechanism provides a natural solution to the strong CP problem [2]. In the mechanism, one introduces an axion [2, 3, 4], which is charged under the PQ symmetry. Under the PQ transformation, the axion field  $a$  gets shifted as  $a \rightarrow a + f_a \epsilon$ , where  $f_a$  denotes the axion decay constant (or the PQ scale), and  $\epsilon$  is the transformation parameter. In what follows we normalize the axion  $a$  by  $f_a$  so that  $a$  is dimensionless. The axion is assumed to be coupled to the QCD anomaly,

$$\mathcal{L} = \frac{g_s^2}{64\pi^2} a \epsilon_{\mu\nu\rho\sigma} G^{(a)\mu\nu} G^{(a)\rho\sigma}. \quad (2)$$

After the QCD phase transition, the axion gets stabilized due to the QCD instanton effect, satisfying  $a + \theta = 0$ . Thus the strong CP problem is solved dynamically.

Since the PQ mechanism was proposed, a lot of efforts have been made to implement the mechanism. The models proposed so far can be divided broadly into two categories. One adopts a field theoretic approach using a  $U(1)_{\text{PQ}}$  symmetry. In the DFSZ [5, 6] and KSVZ (or hadronic) [7, 8] axion models, a global  $U(1)_{\text{PQ}}$  symmetry is introduced, which is spontaneously broken by a vacuum expectation value (VEV) of a scalar field. The associated Nambu-Goldstone boson becomes an axion. Those models fall in this category. The other identifies one of the axion-like fields in the string theory to be the QCD axion. We focus on the latter category throughout this letter.

The string theory is currently the most promising candidate for a unified theory of all forces including gravity [9]. Moreover, it contains many axion-like fields associated with the Green-Schwarz mechanism [10]. Therefore, it is natural to seek for the QCD axion in the string set-up. However, it turns out that there are severe cosmological problems associated with the axion.

The PQ scale  $f_a$  is constrained as  $10^9 \text{ GeV} \lesssim f_a \lesssim 10^{12} \text{ GeV}$  [12, 13, 14] from astrophysical and cosmological considerations. The upper bound comes from the requirement that the axion density should not exceed the observed amount of dark matter (DM), based on an assumption that the initial displacement of the axion from the nearest minimum is  $\mathcal{O}(1)$ . However, the PQ scale is expected to be as large as  $\mathcal{O}(10^{16}) \text{ GeV}$  in the string theory. If  $f_a$  is as large as  $10^{16} \text{ GeV}$ , the axion abundance would exceed the observed DM abundance by many orders of magnitudes. Although we may hope that the axion model with smaller  $f_a$  can be constructed, currently it seems hard to make the value of  $f_a$  much smaller than  $10^{16} \text{ GeV}$  [11]. There are several solutions proposed so far; (i) to dilute the axion abundance by the late-time entropy production [15]; (ii) to set the initial position of the axion very close to the CP conserving minimum. However both are not completely satisfactory.

The first solution (i) is most easily realized by introducing the late-time decaying particle [16] or unstable topological defects [17], which produce enormous amount of the entropy at the decay. However, since the pre-existing baryon asymmetry is also diluted, we have to rely on a very efficient baryogenesis scenario such as the Affleck-Dine mechanism [18, 19, 20, 21, 22, 23]. We do not argue that it is impossible to have consistent cosmology in this case, but the cosmology required by this solution is far from the simplest one, making us feel that it is slightly contrived.

In the second solution (ii), we need to fine-tune the initial position of the axion. Since the axion likely takes a randomly chosen value due to quantum fluctuations during inflation, we need to indeed fine-tune the initial position by hand. One may hope that the initial position of the axion might be selected in such a way that the axion abundance does not exceed the DM abundance [24], based on the anthropic principle. When applied to the

cosmological constant, the anthropic principle was successful as shown in [25]. However, the recent analysis showed that the constraint on the DM abundance, therefore on the initial position of the axion, is too loose based on the simple anthropic argument [26]. On the other hand, the authors of Ref. [27] performed much more detailed studies by taking account of e.g. the comet impact rate in a universe with a larger amount of dark matter. Their results showed that the anthropically favored value of the dark matter abundance is very close to the observed one. While we agree that the comet impact rate can have an important effect on the existence of life, it is not easy to estimate its effect precisely due to our limited knowledge.

The bosonic supersymmetric (SUSY) partner of the axion, saxion, also leads to a severe cosmological problem [28, 29, 30], which is similar to the notorious cosmological moduli problem [31, 32, 33, 34]. One may be able to solve the problem in a similar fashion described above, but the resultant cosmology again does not seem natural. Note also that the anthropic argument on the (s)axion abundance cannot solve the problem, unless the saxion is stable in cosmological time.

While a starting point is well grounded theoretically, i.e., the axion elegantly solves the strong CP problem and the string theory seems to be the plausible candidate to implement the PQ mechanism, we are nevertheless led to either apparently contrived cosmology or the fine-tuning. Those tantalizing situation can be viewed as a hint that we might have made a wrong assumption from the very beginning. That is to say, the dynamical solution to the strong CP problem may not be the correct answer, if the axion is to be embedded in the string theory.

In this letter, we give up the ordinary PQ mechanism, and instead, we consider what happens if the PQ symmetry is explicitly broken other than the QCD instantons. The beauty of the PQ mechanism has prevented most people to pursue this possibility seriously. We find that the CP conserving minimum can be anthropically selected, if the probability distribution of the vacuum energy excluding the contribution from the axion sector has a pressure toward higher values. Whether the probability distribution possesses such a property or not is tied to the cosmological measure problem, and we do not have a definite

answer at the moment. We will, however, give several possibilities that such a feature may appear.

It is quite interesting to note that the axion can acquire a large mass due to the explicit breaking, and it may be absent in the low-energy particle spectrum <sup>a</sup>. This striking feature has rich implications for cosmology. All the cosmological problems associated with the (s)axion are solved, if the (s)axion mass is large enough. The axion may come to dominate the energy density of the universe after inflation, and reheat the universe by its decay. It is even possible to make the cosmological abundance of the axion negligible, if the explicit breaking is large enough during inflation.

To summarize, with our conjecture on the probability distribution of the vacuum energy, we arrive at the followings.

1. The strong CP problem is resolved by the anthropic reasoning.
2. The cosmological problems of the (s)axion with large  $f_a$  can be solved.
3. Interesting cosmological scenarios emerge: the axion may dominate and reheat the universe; the axion may generate the cosmological density perturbations.

In the following sections, we will detail each point.

## II. THE ANTHROPIC SOLUTION TO THE STRONG CP PROBLEM

Now let us explain how it works. The shift symmetry of the axion is violated by the QCD instantons. After the QCD phase transition, the instantons generate the effective potential of the axion,

$$V_{\text{QCD}}(a) = \Lambda_{\text{QCD}}^4 (1 - \cos a), \quad (3)$$

where the axion field  $a$  is dimensionless, and we have chosen the CP conserving minimum to be at  $a = 0$  for simplicity. We drop numerical coefficients of order unity here and in what

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<sup>a</sup> We use the terminology, “axion”, although it is not the ordinary massless QCD axion in the PQ mechanism.

follows, since they are irrelevant for our discussion. If there are no other contributions to the axion potential, the axion will settle down to  $a = 0$  after the QCD phase transition, and the strong CP problem is dynamically solved.

Let us introduce another explicit breaking of the shift symmetry, which generates the following potential,

$$V_{inst}(a) = \Lambda_{inst}^4 (1 - \cos(a - \psi)) \quad (4)$$

where  $\psi$  denotes the minimum of the explicit breaking term. Indeed, there is such breaking of the shift symmetry due to some sort of the instantons in the string theory [11]. The precise form of the explicit breaking is not important here. How the explicit breaking is generated and how large it is will be discussed later. For the moment we assume that the potential  $V_{inst}$  is the only source for the explicit breaking of the shift symmetry, other than the QCD instanton. The total axion potential after the QCD phase transition is given by  $V(a) = V_{QCD}(a) + V_{inst}(a)$ . See Fig. 1. We assume that the breaking term is much larger than the term arising from the QCD instantons, i.e.,

$$\Lambda_{inst} \gg \Lambda_{QCD}. \quad (5)$$

Then the minimum of the axion potential  $V(a)$  is essentially determined by that of  $V_{inst}(a)$ . That is,  $V(a)$  takes the minimal value at  $a \approx \psi$ . Generically we expect  $\psi = \mathcal{O}(1)$ , because there is no a priori reason for the explicit breaking term to have its minimum just at the CP conserving one. Therefore, we have intolerably large CP phase in the presence of the large explicit breaking of the PQ symmetry, as expected. That is why we usually assume that such explicit breaking is somehow suppressed for the PQ mechanism to work.

We now assume that  $\psi$  is an environmental variable, which takes different values in different regions in the universe that are separated far apart from one another. One can imagine a situation that there are an infinitely large number of expanding regions, in each of which  $\psi$  takes a different value.

We note that the axion potential at the minimum becomes the smallest when  $\psi$  equals to 0, i.e., when the minimum of  $V_{inst}$  happens to coincide with that of  $V_{QCD}$  (see the bottom panel in Fig. 1). In this sense the CP conserving minimum is special. However,

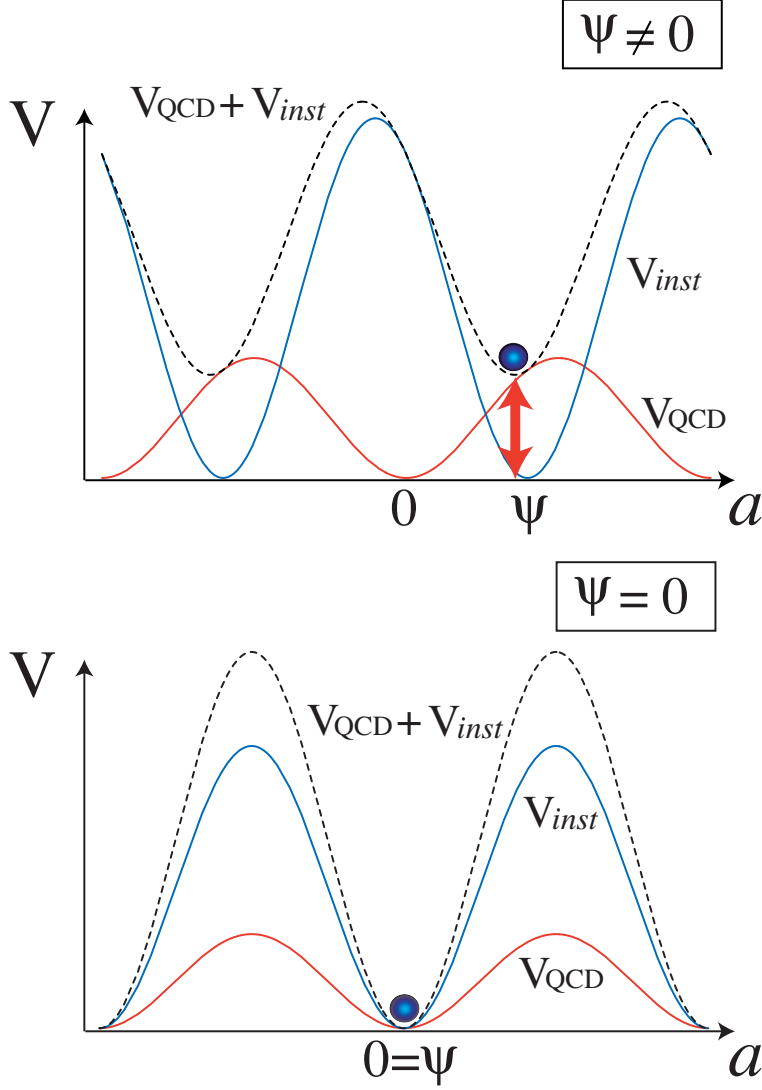


FIG. 1: The axion potentials,  $V_{\text{QCD}}$ ,  $V_{\text{inst}}$ , and  $V_{\text{QCD}} + V_{\text{inst}}$ , for  $\psi \neq 0$  (top) and  $\psi = 0$  (bottom). The circle represents the minimum of the axion potential, and the arrow shows non-zero cosmological constant at the minimum.

since there are many other contributions to the total cosmological constant, one cannot naively argue that the CP conserving minimum with  $\psi = 0$  should be selected simply because it minimizes the contribution from the axion sector.

To illustrate our idea, let us consider such a universe with  $\psi = 0$  that the total cosmo-

logical constant including all the possible contributions is within the anthropic window, i.e., the cosmological constant is small enough to make the universe habitable. Below we will discuss a condition that such a universe becomes more likely than the others. For the moment, let us consider what happens if we vary  $\psi$  from 0 in such a universe. Then, even if  $\psi$  were slightly different from 0, the cosmological constant would be greatly enhanced as  $\rho_{\text{cc}} \approx \Lambda_{\text{QCD}}^4 |\psi|^2$  for  $|\psi| \ll 1$ , and it would be out of the anthropic window. Here  $\rho_{\text{cc}}$  denotes the energy density of the cosmological constant, including all the contributions. Thus, life cannot arise in such universe with  $\psi = \mathcal{O}(1)$ , and the universe with almost vanishing  $\psi$  will be selected by the anthropic principle. It is worth mentioning here that the explicit breaking of the PQ symmetry has reduced the strong CP problem to the cosmological constant problem.

The remaining issue is why the total cosmological constant should be almost zero (more precisely, within the anthropic window) in the universe with  $|\psi| \approx 0$ . In other words, among those universes with the total cosmological constant satisfying the anthropic bound, is there any reason to favor smaller values of  $|\psi|$ ? We here adopt a conjecture that there are infinitely large number of meta-stable vacua, in each of which the cosmological constant takes a variety of values, i.e., the so-called string landscape [35, 36, 37]. To be explicit, we express the energy density of the cosmological constant as follows:

$$\rho_{\text{cc}} = \rho_L + \rho_{\text{axion}}(\psi) \quad (6)$$

with

$$\rho_{\text{axion}}(\psi) \equiv V(a)|_{a=\psi}. \quad (7)$$

The first term in Eq. (6),  $\rho_L$ , is supposed to contain all the contributions such as the quantum corrections, the electroweak symmetry breaking and the string landscape, except for the axion potential, which is represented by the second term,  $\rho_{\text{axion}}(\psi)$ .  $\rho_L$  can take a variety of values in the huge number of vacua, which enables us to live in such a universe that the first and second terms (almost) cancel with each other, giving  $\rho_{\text{cc}} \approx 0$ . In the following we assume that the main effects of varying  $\psi$  is to change  $V(a)$ , i.e.,  $\rho_{\text{axion}}(\psi)$ . More precisely, if we change  $\psi$  with all the other parameters being fixed, the change in  $\rho_{\text{cc}}$  is assumed to be dominantly given by the change in  $\rho_{\text{axion}}(\psi)$ .



One of the interesting features of the string landscape is that one can in principle quantify the naturalness in terms of probability by e.g. counting the number and/or weighing the volume of the vacua satisfying certain conditions of interest [38, 39]. Let us define the probability distribution  $P_L(\rho_L)$  in such a way that the probability that a vacuum has  $\rho_L$  in the range of  $\rho_L \sim \rho_L + \Delta\rho_L$  is given by  $P_L(\rho_L)\Delta\rho_L$ . The probability is assumed to include not only the a priori probability distribution, but also the other effects such as the volume due to the eternal inflation and the statistical (or dynamical) properties of scanning the landscape.

We assume that the probability distribution of  $\psi$  is flat for simplicity. Then the resultant probability distribution of  $\rho_{\text{axion}}$  is given by

$$P_{\text{axion}}(\rho_{\text{axion}}) = \begin{cases} \frac{1}{\pi\Lambda_{\text{QCD}}^4} \left[ 2 \left( \frac{\rho_{\text{axion}}}{\Lambda_{\text{QCD}}^4} \right) - \left( \frac{\rho_{\text{axion}}}{\Lambda_{\text{QCD}}^4} \right)^2 \right]^{-\frac{1}{2}} & \text{for } 0 < \rho_{\text{axion}} < 2\Lambda_{\text{QCD}}^4, \\ 0 & \text{otherwise.} \end{cases} \quad (8)$$

Note that the maximum and minimum are not specially favored, although  $P_{\text{axion}}(\rho_{\text{axion}})$  diverges at  $\rho_{\text{axion}} = 0$  and  $2\Lambda_{\text{QCD}}^4$ ; the probability remains finite.

The total cosmological constant  $\rho_{\text{cc}}$  is given by the sum of  $\rho_L$  and  $\rho_{\text{axion}}$ , as shown in Eq. (6). Therefore we naively expect that there are many ways to make the first and the second terms almost cancel with each other so that the total cosmological constant  $\rho_{\text{cc}}$  is within the anthropic window,  $0 < \rho_{\text{cc}} \lesssim \rho_{\text{cc}}^{(\text{aw})} = \mathcal{O}((1 \text{ meV})^4)$ .

First, let us consider a case that  $P_L(\rho_L)$  is independent of  $\rho_L$  over an interested range of  $\rho_L$ . We call this case as the flat distribution. Then, a vacuum satisfying  $0 < \rho_{\text{cc}} \lesssim \rho_{\text{cc}}^{(\text{aw})}$  does not favor any particular value of  $\psi$ . Whatever value  $\psi$  takes, there are some fixed number of vacua that makes  $\rho_{\text{cc}}$  almost zero. In this sense, the universe with  $\psi = 0$  is as likely as that with e.g.  $\psi = 1$ . Therefore, if the probability distribution of  $\rho_L$  is flat over an interested range of  $\rho_L$ , one cannot solve the strong CP problem by the anthropic reasoning.

The situation greatly changes if we allow  $P_L(\rho_L)$  to depend on  $\rho_L$ . Suppose that  $P_L(\rho_L)$  grows as  $\rho_L$  increases. We call this case as the steep distribution. For instance, we can

imagine an exponential form,  $P_L(\rho_L) = P_0 \exp(\rho_L/\rho_0)$ . See Fig. 2. Then, among those vacua satisfying the anthropic bound,  $0 < \rho_{cc} \lesssim \rho_{cc}^{(\text{aw})}$ , smaller values of  $|\psi|$  are favored, i.e., the universe with  $\psi \approx 0$  is more likely than that with e.g.  $\psi = 1$ . This is simply because the probability distribution  $P_L(\rho_L)$  is enhanced as  $\rho_L$  increases, i.e., as  $\rho_{\text{axion}}$  decreases. Note that the anthropic bound requires

$$0 < \rho_L + \rho_{\text{axion}} \lesssim \rho_{cc}^{(\text{aw})}. \quad (9)$$

With this bound satisfied, making  $\rho_L$  larger is equivalent to making  $\rho_{\text{axion}}$  smaller. Due to the steep distribution, larger  $\rho_L$ , or equivalently, smaller  $\rho_{\text{axion}}$  is favored. Thus, if the probability distribution of  $\rho_L$  is steep enough, the universe with  $\psi \approx 0$  is statistically favored among the universes satisfying the anthropic constraint on the cosmological constant.

Let us evaluate how steep  $P_L(\rho_L)$  should be to solve the strong CP problem. To satisfy the current bound on  $\theta$ ,  $\psi$  must be constrained as  $|\psi| < \theta^{(\text{exp})} = 10^{-(9-10)}$ . For  $|\psi| \ll 1$ , we have approximately

$$\rho_{\text{axion}}(\psi) \simeq V_{\text{QCD}}(\psi) \simeq \Lambda_{\text{QCD}}^4 \frac{\psi^2}{2}. \quad (10)$$

In order to statistically favor the universe with  $|\psi| < \theta^{(\text{exp})}$  over the universe with  $|\psi| > \theta^{(\text{exp})}$ , the following condition must be met;

$$\int_{\rho_{cc}^{(\text{aw})} - \frac{1}{2}\theta^{(\text{exp})2}\Lambda_{\text{QCD}}^4}^{\rho_{cc}^{(\text{aw})}} P_L(\rho_L) d\rho_L \gtrsim \int_{\rho_{cc}^{(\text{aw})} - \Lambda_{\text{QCD}}^4}^{\rho_{cc}^{(\text{aw})} - \frac{1}{2}\theta^{(\text{exp})2}\Lambda_{\text{QCD}}^4} P_L(\rho_L) d\rho_L. \quad (11)$$

If one adopts an exponential form,  $P_L(\rho_L) = P_0 \exp(\rho_L/\rho_0)$ , the condition amounts to

$$\rho_0 \lesssim \theta^{(\text{exp})2} \Lambda_{\text{QCD}}^4 \approx (1 \text{ keV})^4. \quad (12)$$

We would like to emphasize that such a steep distribution does not have to persist over an entire range of  $\rho_L$ . If the probability distribution is locally steep at  $\rho_L \approx \rho_{cc}^{(\text{aw})} - \rho_{\text{axion}}$  over a range of  $\sim \Lambda_{\text{QCD}}^4$ , our arguments above is valid <sup>b</sup>.

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<sup>b</sup> In the presence of multiple breaking terms with the strengths,  $\Lambda_1^4 \ll \dots \ll \Lambda_{n-1}^4 \ll \Lambda_n^4$ , with the relative differences of the minima being the environmental variables, we need to assume that the steep distribution persists at least over a range of  $\Lambda_{n-1}^4$ .

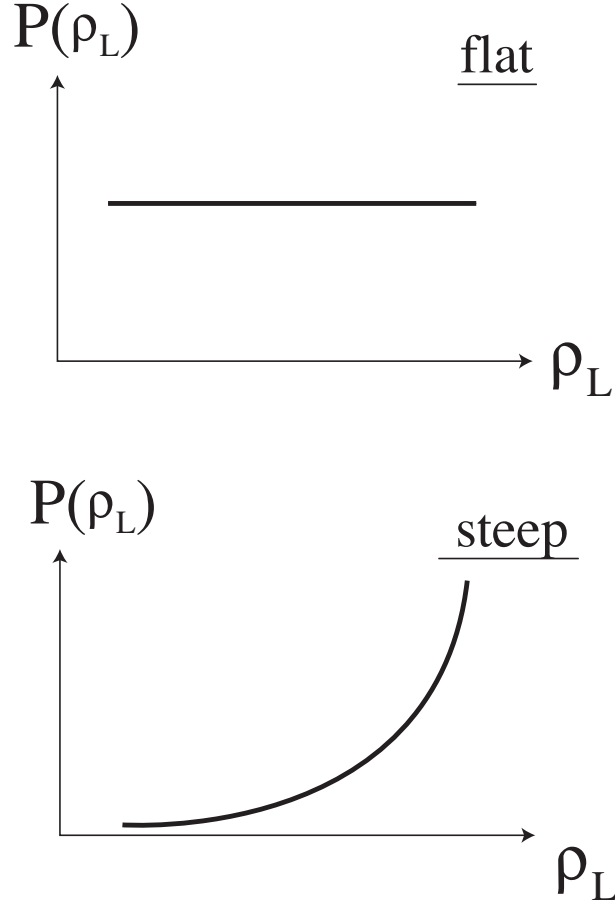


FIG. 2: The probability distributions of  $\rho_L$ . The flat (top) and steep (bottom) distributions are shown. The range of the distribution shown in this figure is supposed to be at least  $\sim \Lambda_{\text{QCD}}^4$ . We do not need to assume global behavior of  $P_L$ . (see also footnote b)

Several remarks are as follows. We have assumed that the probability distribution of  $\psi$  is almost flat. However, it is not necessarily flat; it can depend on  $\psi$  as long as the dependence is mild enough that the argument above using the steepness in  $P_L$  remains valid. We can also imagine that  $\theta$  as well may be an environmental variable. In this case, the above argument remains unchanged by simply replacing  $\psi$  with  $\psi' \equiv \psi - \theta$ . If there are multiple scalars that couple to the QCD anomaly, we take our axion as the lightest one, while the others are integrated out. Then the possible effects of the heavier particles

can be represented by varying  $\theta$ . One may wonder if the axion is not introduced from the beginning but  $\theta$  is still regarded as an environmental parameter. One can reach the same conclusion, since this essentially corresponds to the case that the axion is integrated out.

One may worry that the required steepness in the probability distribution of  $\rho_L$  may contradict with the flat prior that is usually assumed when one applies the anthropic principle to the cosmological constant problem. Both can be reconciled if the steepness is rather weak over the typical value of the cosmological constant within the anthropic window, but still strong enough to select the universe with  $\psi \approx 0$ . In the case of the exponential form, this is satisfied if

$$\rho_{\text{cc}}^{(\text{aw})} \ll \rho_0 \lesssim \theta^{(\text{exp})2} \Lambda_{\text{QCD}}^4, \quad (13)$$

or equivalently,  $(1 \text{ meV})^4 \ll \rho_0 \lesssim (1 \text{ keV})^4$ , is met.

What is the possible origin of the hierarchical distribution of  $\rho_L$ ? Such a steep distribution is just a trade-off with the fine-tuning of the initial position of the axion, until we find its origin. Interestingly, however, there are several proposals that the probability distribution of  $\rho_L$  might differ from the flat distribution [40, 41, 42, 43, 44, 45]. In any case, this is closely related to the cosmological measure problem, which is not settled yet <sup>c</sup>.

Here we give one possible explanation for the steep distribution. We assume that there are many meta-stable vacua with different values of the cosmological constant. The universe trapped in a false vacuum with a large cosmological constant will experience eternal inflation. After a long time, a bubble will be created with its center at a vacuum with smaller cosmological constant. One might expect that the most rapidly-inflating vacuum gives the dominant probability, since it has an exponentially large volume. Although it is not easy to define the gauge-invariant measure that rewards the volume, we here assume that such a measure can be defined properly. Note that the QCD instanton effect is suppressed before the QCD phase transition. Therefore, in an epoch much before the QCD phase transition, there is a difference in the energy density between the vacuum “A” with  $\psi = 0$  and the vacuum “B” with  $\psi = 10^{-9}$ , as long as both satisfy the anthropic bound

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<sup>c</sup> We thank R. Bousso for discussion and the healthy criticism of such steep distribution.

(9). The energy difference,  $\rho_A - \rho_B$ , will be of  $\mathcal{O}(\text{keV}^4)$ . We assume that all the other parameters other than  $\psi$  are fixed. The energy scale of  $\mathcal{O}(\text{keV}^4)$  is small compared to the energy density of the universe before BBN, which makes difficult to distinguish the two vacua  $A$  and  $B$  by the ordinary cosmological evolution. Suppose that there are first order phase transitions from the long-lived meta-stable vacua,  $A'$  and  $B'$  into the vacua  $A$  and  $B$ , respectively. The energy densities of  $A'$  and  $B'$  are denoted by  $\rho_{A'}$  and  $\rho_{B'}$ . We are considering such a situation that the vacuum  $A'(B')$  is adjacent to the vacuum  $A(B)$ , while the vacua  $A'$  and  $B'$ , therefore  $A$  and  $B$ , are far apart from each other. We assume that a typical (or averaged) energy difference between the vacua  $A$  and  $A'$  is equal to that between  $B$  and  $B'$ , since  $\mathcal{O}(\text{keV}^4)$  is so small compared to the fundamental scale. It means that  $\rho_{A'}$  tends to be slightly larger than  $\rho_{B'}$  by  $\mathcal{O}(\text{keV}^4)$ . The difference of the energy densities,  $\rho_{A'} - \rho_A$  as well as  $\rho_{B'} - \rho_B$ , takes a variety of values with probably large dispersion. So, one has to collect at least  $((M_P/\text{keV})^4)^2 \sim 10^{195}$  vacua, in order to see such a tiny difference. Of course, since  $\mathcal{O}(\text{keV}^4)$  is much smaller than the fundamental scale, it does not affect the cosmological expansion in most cases. However, if the meta-stable vacua  $A'$  and  $B'$  are very long-lived, say, if the typical number of the e-foldings during the inflation in the vacua  $A'$  and  $B'$  is exponentially large, such a tiny difference in the energy density may result in significant difference in the final volume. Thus, we may have a steep distribution  $P_L$ , which changes over a scale of  $\mathcal{O}(\text{keV}^4)$ . Note that, in the above explanation, the origin of the steepness is the huge number of the vacua and the longevity of the meta-stable vacua  $A'$  and  $B'$ .

So far we have assumed that the explicit breaking is much larger than the QCD instanton effects. For our arguments to be valid, the axion abundance should not contribute to the DM abundance. If it does, we need to perform analysis along the line of Ref. [26], and we will typically end up with the DM abundance much larger than the observed value. Therefore the explicit breaking is assumed to be large enough that the axion does not contribute to the DM abundance. If there are several explicit breaking terms with different strength, this restriction on the size applies to the largest one. The anthropic argument can be similarly applied to the smaller breaking terms. In particular it is no problem

to apply to the breaking terms smaller than the QCD instanton effects, as long as the breaking is much larger than  $\sim (1\text{meV})^4$ .

### III. COSMOLOGY

In the ordinary PQ mechanism, the axion acquires its mass mainly from the QCD instanton effects represented by (3), and the mass is given by

$$m_a \sim m_\pi \frac{F_\pi}{f_a} \simeq 1 \times 10^{-9} \text{ eV} \left( \frac{f_a}{10^{16} \text{ GeV}} \right)^{-1}, \quad (14)$$

where the numerical coefficient weakly depends on the axion models. Thus the axion is usually very light and stable, and that is why the axion is one of the candidates for the DM. In our scenario, however, the axion acquires a large mass due to the explicit breaking of the PQ symmetry. Assuming the breaking term given by (4), the axion mass is

$$m_a \sim \frac{\Lambda_{inst}^2}{f_a}. \quad (15)$$

So, the axion mass sensitively depends on  $\Lambda_{inst}$ .

How large is  $\Lambda_{inst}$ ? In the string theory, there are several sources for the explicit breaking of the shift symmetry: the world-sheet instantons, brane instantons, gauge instantons from other factors of the gauge group, and gravitational instantons [11]. Since all of them are non-perturbative effects,  $\Lambda_{inst}$  can be exponentially suppressed relative to the fundamental scale,  $M$ , which can be as large as the reduced Planck scale,  $M_P = 2.4 \times 10^{18} \text{ GeV}$ . That is, we estimate  $\Lambda_{inst}^4 = M^4 \exp(-S_{inst})$ , where  $S_{inst}$  denotes the action of the instanton. Or, in the presence of low-energy SUSY, it might be further suppressed as  $\Lambda_{inst}^4 = M^2 \mu^2 \exp(-S_{inst})$ , where  $\mu = \sqrt{m_{3/2} M_P}$  is the SUSY breaking scale. In order to have the successful PQ mechanism, it is usually assumed that the action  $S_{inst}$  is very large (e.g.  $S \simeq 200$ ), which suppresses the explicit breakings small enough. For our purpose,  $S$  should not be that large, since we need the large explicit breaking terms. Since the size of the breaking  $\Lambda_{inst}^4$  is very sensitive to  $S_{inst}$ , it is important to estimate the value of  $S_{inst}$  very precisely. We here simply treat  $\Lambda_{inst}$  (therefore  $m_a$ ) as a free parameter.

First let us consider a case that the axion mass is heavier than the cosmic expansion rate during inflation, i.e.,  $m_a > H_{inf}$ . Then the axion settles down to the potential minimum during inflation. Since the anthropic argument requires the minimum to coincide with the CP conserving one, the axion remains to stay there after inflation, and the cosmological abundance of the axion is negligible. Therefore, in this case, the axion does not play any important role in cosmology.

Next we take up the other case that the axion mass is lighter than the cosmic expansion rate during inflation. Then the position of the axion during inflation is expected to be away from the CP conserving minimum by  $\mathcal{O}(1)$ . After inflation, the axion starts to oscillate when the Hubble parameter becomes comparable to the axion mass. What is different from the ordinary PQ mechanism is that the oscillations can start in much earlier phase of the universe, and more importantly, that the axion is unstable and decays into the SM particles.

The partial decay rate of the axion into a pair of the gluons through (2) is given by

$$\Gamma(a \rightarrow 2g) \simeq \frac{\alpha_s^2}{64\pi^3} \frac{m_a^3}{f_a^2}. \quad (16)$$

Assuming that the possible decay processes into the other sectors are kinematically forbidden, the decay temperature of the axion,  $T_a$ , is

$$T_a \simeq 8 \times 10^7 \text{ GeV} \left( \frac{g_*}{200} \right)^{-\frac{1}{4}} \left( \frac{\alpha_s}{0.05} \right) \left( \frac{m_a}{10^{12} \text{ GeV}} \right)^{\frac{3}{2}} \left( \frac{f_a}{10^{16} \text{ GeV}} \right)^{-1}, \quad (17)$$

where  $g_*$  counts the relativistic degrees of freedom at the decay. Since the initial amplitude of the axion is as large as  $f_a = \mathcal{O}(10^{16}) \text{ GeV}$ , the axion abundance tends to be quite large. Therefore the axion must decay before the big bang nucleosynthesis (BBN) starts. Requiring  $T_a \gtrsim 10 \text{ MeV}$  [46], the axion mass is bounded below:

$$m_a \gtrsim 2 \times 10^5 \text{ GeV} \left( \frac{g_*}{200} \right)^{\frac{1}{6}} \left( \frac{\alpha_s}{0.05} \right)^{-\frac{2}{3}} \left( \frac{f_a}{10^{16} \text{ GeV}} \right)^{\frac{2}{3}}. \quad (18)$$

The explicit breaking of the shift symmetry should be large enough that this condition is met when  $m_a \lesssim H_{inf}$ . Note that the lower limit is not applied in the case of  $m_a \gtrsim H_{inf}$ .

The cosmological abundance of the axion is estimated to be

$$\frac{\rho_a}{s} \simeq \frac{1}{8} T_{inf} \left( \frac{f_a}{M_P} \right)^2, \quad (19)$$

where  $\rho_a$  is the energy density of the axion,  $s$  the entropy density, and  $T_{inf}$  the inflaton decay temperature. We have here assumed that the initial displacement of the axion from the minimum is equal to 1, and that the axion does not dominate the energy density of the universe. This is the case if  $T_{inf} \lesssim 6T_a(M_P/f_a)^2$ . On the other hand, if  $T_{inf} \gtrsim 6T_a(M_P/f_a)^2$ , the axion dominates the universe, and the (last) reheating of the universe is provided by the decay of the axion.

The latter possibility is particularly interesting. The reheating temperature of the universe is completely determined by the parameters of the axion sector, i.e.,  $\Lambda_{inst}$  and  $f_a$ , which are in principle calculable once the axion model is fixed in the string theory. Furthermore, since the axion is light during inflation, it acquires quantum fluctuations, which turn into the adiabatic density perturbations after the decay <sup>d</sup>. That is, the axion can be a curvaton [47], if the inflation scale is  $H_{inf} \sim 10^{-5}(2\pi f_a) \sim 10^{12}$  GeV. Such a paradigm may help us to construct an inflation model in the stringy set-up, because the density perturbations do not have to be generated by the inflaton, and because the reheating is naturally induced by the axion that has a couplings to the SM sector.

We make several remarks on the other cosmological implications. Note that, since there is no need to dilute the axion, the attractive cosmological scenarios such as the leptogenesis [51] are feasible for large enough  $m_a$ . Even if the axion mass is relatively small and the reheating temperature due to the axion decay becomes rather low, we do not need to introduce another sector in order to dilute the axion. In our scenario, the axion does not contribute to the DM abundance, which suggests that other candidates such as WIMP and the gravitino should account for the DM. The failure of the anthropic argument to account for both the cosmological constant and the DM abundance simultaneously may hint that the DM abundance is determined by the physics, not by the anthropic reasoning. Also, since the saxion mass is also large, its cosmological problem can be solved in a similar fashion. In the discussion above, we have simply assumed that the axion mainly decays into a pair of the gluons. There might be other decay processes at tree-level [32, 34, 52, 53]

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<sup>d</sup> If the axion does not dominate the energy density of the universe, it may be able to generate large non-Gaussianity either by the curvaton mechanism [48] or by the ungaussiton mechanism [49, 50].



as well as one-loop level [54]. In particular, the gravitino might be non-thermally produced by the axion decay, which may help us further constrain the axion models.

#### IV. CONCLUSIONS AND DISCUSSION

The existence of the axion-like fields is quite common in the string theory. They generically receive explicit breakings of the shift symmetries due to the world-sheet instantons, brane instantons, gauge instantons from other factors of the gauge group, and gravitational instantons. It was indeed an issue how to suppress such explicit breakings in order to have the successful PQ mechanism [11]. This generically set a restriction on the theory. We have offered a possibility to solve the strong CP problem in the presence of large explicit breaking terms, which therefore liberate the theory from such restriction. An essential ingredient is an assumption that the probability distribution of the vacuum energy excluding the contribution from the axion sector has a pressure towards higher values. Then, among those vacua satisfying the anthropic bound on the cosmological constant, the CP conserving minimum is statistically favored since it minimizes the contribution from the axion sector. Note that such a probability distribution can be consistent with a flat prior usually assumed when one applies the anthropic principle to the cosmological constant problem, since the energy scales of interest are different.

At present we do not know the origin of the steep probability distribution. It is interesting to note, however, that there are some proposals that the distribution might differ from the flat distribution [40, 41, 42, 43]. The source of the hierarchy may be the statistical property of the scanning of  $\rho_L$  and/or some dynamics such as the bubble nucleation. The measure of the distribution of vacua, taking account of the cosmic expansion during eternal inflation, may also help us to understand the origin of such hierarchy.

It would be encouraging if we can find other examples in which such steep vacuum distribution plays an important role to determine physical environmental parameters. In particular, if the the probability distribution is steep over a scale of the weak scale or larger, it will favor a heavier Higgs mass, since it results in the deeper potential well for

the fixed Higgs VEV. Thus, it will be quite interesting and suggestive, if the little hierarchy problem associated with the Higgs mass can be interpreted as the result of such vacuum distribution. Of course we need to properly take account of the anthropic window on the electroweak breaking scale, in order to claim that the steep vacuum distribution favors large one-loop corrections to the Higgs mass.

Throughout this letter we have not specified the source for the explicit breaking. If it is large enough, the axion will settle down at the CP conserving minimum during inflation. Thus the cosmological abundance of the axion is negligible in this case. It is also possible that the axion dominates the energy density of the universe after inflation and reheats the universe by the decay, if the explicit breaking is relatively small during inflation.

How large the explicit breaking can be in the realistic string theory and its implication on the inflationary scale are very interesting issues, and we leave them for future work. Whether a steep probability distribution is indeed feasible or not, as well as how much hierarchy can be realized and from what it is originated, are open questions. Hopefully, future development in the string theory and the associated areas may enable us to answer all or some of these questions.

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